ME 536   Comput Intell in Control Engr

Homework 6:   Deadline: May 15, 2020

Course Value: 100 points

F-150 trailer back-up assist reverse engineering through feedback linearization.

The 'bicycle model' of a car backing up with a trailer in tow is illustrated below. For convenience, we define the ground reference frame as positive x to the right and positive y 'up'. The body fixed axis is then defined as x pointing in the direction of rear axis travel, and y pointing to the driver's right.

The car 'heading angle' is defined counter-clockwise from the ground x axis, and denoted by ψ. The steering angle is defined counter clockwise from the body negative x axis. The dimensions given below are based on a 1998 GMC Savana towing a 30 ft trailer. For simplicity, we assume that the wheels are located at the corners of the vehicle body.
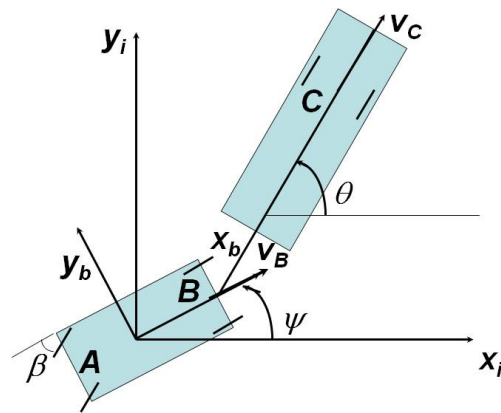
The rear axle is assumed to have a constant velocity, resulting in the equations for velocity of point B.

$$\dot{x}_B = v \cdot \cos \psi$$

$$\dot{y}_B = v \cdot \sin \psi$$

The y component of velocity of point A at the center of the 'front' axle is given by:

$$\dot{y}_A = \frac{v}{\cos \beta} \sin(\psi + \beta)$$

The x component of velocity at point A is then computed from the constraint that the length of the vehicle must remain constant.

$$\dot{x}_A = \tan \psi \, (\dot{y}_B - \dot{y}_A) + \dot{x}_B$$

The car heading angle equation is:

$$\dot{\psi} = -\frac{v}{L} \tan(\beta)$$

The trailer axle speed is given by

$$V_t = v \cos (\psi - \theta)$$

The trailer heading angle is governed by

$$\dot{\theta} = \frac{v}{L_t} \sin(\theta - \psi) \tag{1}$$

and

$$\begin{Bmatrix} \dot{x}_C \\ \dot{y}_C \end{Bmatrix}_{\mathbf{I}} = V_t \begin{Bmatrix} \cos \theta \\ \sin \theta \end{Bmatrix} \tag{2}$$

a) Attempt a feedback linearization design to track a desired trailer cross range (y_c), such that the controller inputs are cross range error and cross range rate error, and the output is the tangent of steering angle ($-0.5 \leq \tan(\beta) \leq +0.5$). Explain why this will not work.

b) Design a feedback linearization controller to track a desired trailer heading (θ), such that the controller inputs are heading error and heading rate error, and the output is the tangent of steering angle ($-0.5 \leq \tan(\beta) \leq +0.5$). Thus, the dynamics as given are *affine* in the control. Choose the linear feedback gains for a natural frequency of 0.232 and a damping ratio of 1. You will need to build the Simulink shown below, replacing the subsystem block with the trailer back-up dynamics from Homework set 1. **Be sure to limit the commanded steering angle as previously stated.** Your Feedback linearization controller block must output steering angle (β) that is, the arctangent must be applied inside this block. **Also your feedback linearization controller may not use heading angle rate ($\dot{\theta}$)—you must substitute in terms of other states!** Note that the controller has forward speed, v, car heading angle ψ, and trailer heading angle θ, as additional inputs. Also note that the *second* output of the error computation block is trailer heading error. Finally, note that the rad/deg and deg/rad gains are set to one since the new controller will work exclusively in radians.
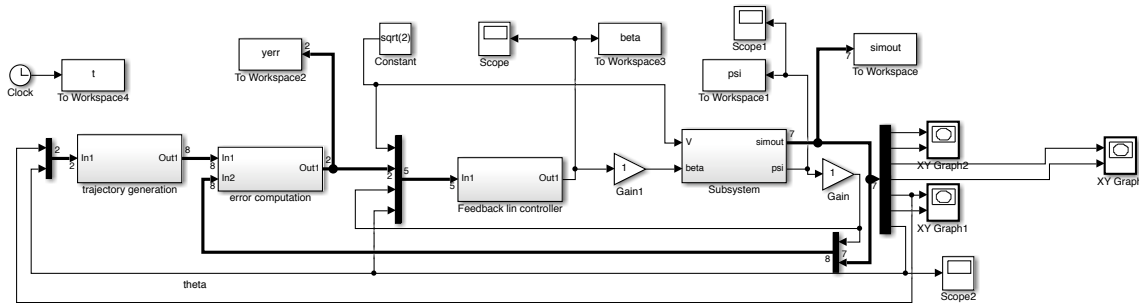
Trajectory generation and error computation block contents are shown below.

a) Trajectory generation

b) Error computation
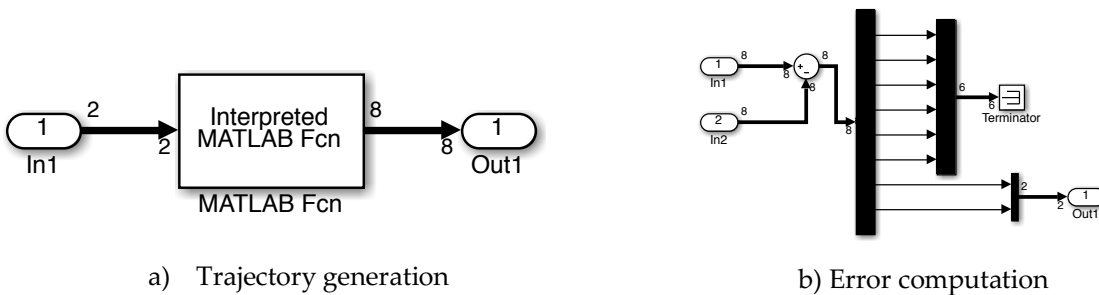
The trajectory generation block contains the matlab function 'cartrailertraj.m' which is linked on the course homework page. Demonstrate performance of your controller by plotting the animation given in carplot_trailer.m (An updated version is linked on the course homework page.)

c) Design a second controller that uses both crossrange and heading error as inputs. Note that this can be done simply by including a small non-zero gain on the cross range error (*first* error computation output). Compare the performance with your previous controller.