

a) (25 points) GA optimization of an integer function: Write a matlab program to optimize $f(x)=x^2$ over the integers $[0, 31]$. This problem was used extensively in class. This exercise is intended to get you started with GA programming.

b) (75 points) GA Optimization of formation measurement topologies using the Calyer-Menger score. Suppose a formation of 10 drones is determining member locations through *cooperative navigation* using intra-formation slant-range measurements. In order to fix the location and pose of the formation, four members are chosen as ‘leaders’. Each of these leaders measures its distance to a known ground station, and provides slant range measurements to some subset of the formation. In order to optimize the accuracy of cooperative navigation by this method, the four leaders must define the vertices of a non-defective tetrahedron in space. That is, they must not all lie in a common plane or nearly in a common plane. Also, to maximize accuracy of navigation, the volume of the tetrahedron should be maximized on average as the formation flies its mission.

You are to write a modified GA that will choose the optimal combination of ‘leaders’ for a formation of 10 drones. Use binary encoding by setting four (and only four) bits high in each 10 bit solution. Modify your mutation code, such that the number of high (‘1’) bits in any candidate solution remains constant at four. Note that crossover may create individuals with more or less than four high bits, and your mutation function should correct each individual to exactly four high bits.

Trajectory information is included in the data file ‘rho.mat’ which is provided. This file contains a comprehensive set of slant range distances for 30 waypoints along the mission trajectory of the formation. Each layer of the 3D array rho contains all possible slant range distances at that waypoint, i.e.:

$$\rho_{i,j} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2}$$

In order for a particular set of drones to define a non-defective tetrahedron, the slant range distances must pass two tests. First the set must be *facial*. Given the example below, #7 is designated the ‘principle’, and $\{1, 2, 4\}$ are ‘wingmen’. Use indexing to map the appropriate $\rho_{i,j}$ to $\{a, b, c\}$ and their complements to

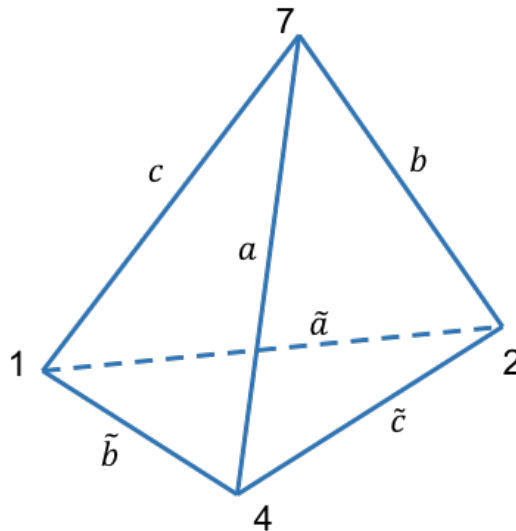


Figure 1: Tetrahedron definitions

$\{\tilde{a}, \tilde{b}, \tilde{c}\}$. The set is facial if:

$$\begin{aligned} \min(a + b + c, a + \tilde{b} + \tilde{c}, \tilde{a} + b + \tilde{c}, \tilde{a} + \tilde{b} + c) \\ > \max(a + \tilde{a}, \tilde{b} + b, \tilde{c} + c) \end{aligned}$$

If a scheme passes the ‘facial’ test, the volume of the tetrahedron is proportional to the Calyer-Menger determinant given by:

$$\begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & a^2 & b^2 & c^2 \\ 1 & a^2 & 0 & \tilde{c}^2 & \tilde{b}^2 \\ 1 & b^2 & \tilde{c}^2 & 0 & \tilde{a}^2 \\ 1 & c^2 & \tilde{b}^2 & \tilde{a}^2 & 0 \end{vmatrix} > 0$$

In order to score a scheme, you should—at each waypoint—assign a logical to denote passage of the facial test, and a real scalar for the Calyer-Menger determinant. The performance score to a particular scheme is the smallest product of the two quantities over the waypoints 2 through 29 (ignore waypoints 1 and 30, and take the minimum of the others, not an average). Schemes that fail the facial test get a score of zero.

Use a population size of 30, and 150 generations to optimize the navigation scheme (select the best set of 4 ‘leaders’). You should reach the point of diminishing returns after 10 generations. The plot below shows the Calyer-Menger scores for all 210 possible combinations. Note the raggedness of the function, which makes it a good candidate for a GA search.

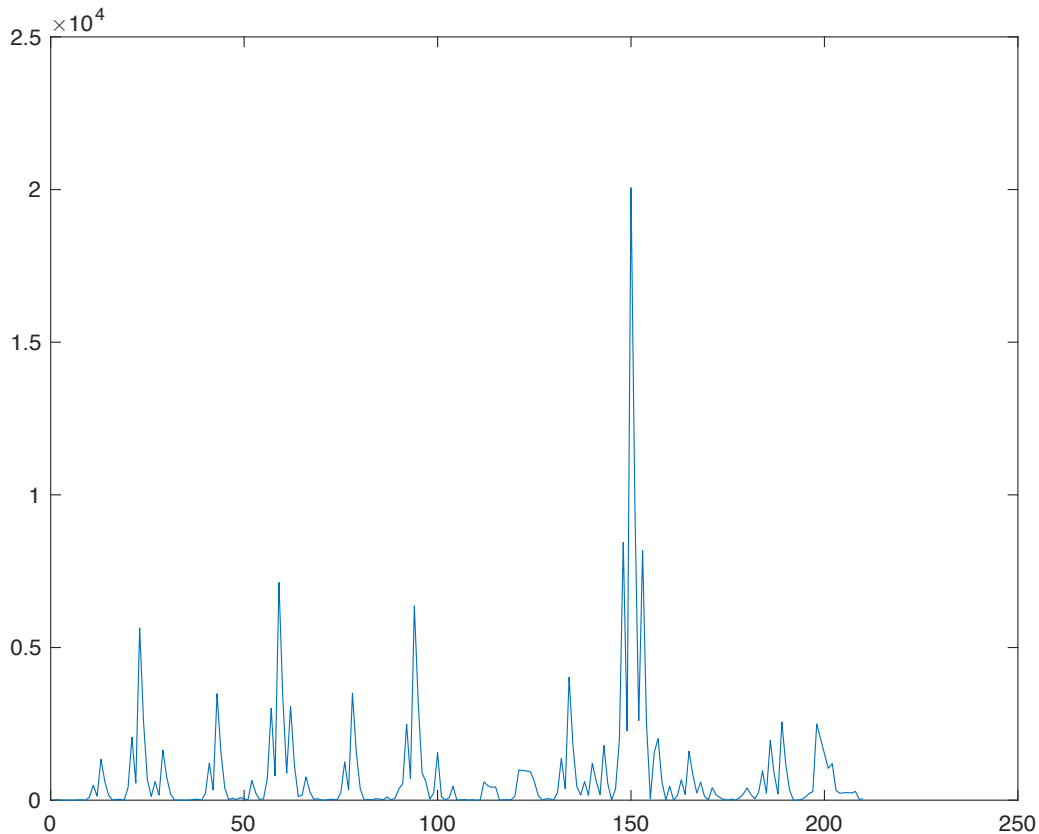


Figure 2. Calyer-Menger score of the 210 possible tetrahedrons.