ME 536   Comput Intell in Control Engr

Homework 1:   Deadline: March 20, 2020
Course Value: 100 points

a) Numerical methods Review I: Write a set of matlab files to simulate the swinging pendulum (from ES214 & 205) using Runge-Kutta 4. Use a time step of 4 ms. Review your notes from ME323/7 for the RK4 algorithm. For this part, integrate Eq. (1) below forward in time for 0.3 sec. Define the initial angle and angular velocity, final time, and $L_{w,cg}$ in the first 3 lines of your script so it is easy for the user to adjust these values. Plot the time history of pendulum swing angle if it is released from rest at $\theta_0 = -\pi/2$ rad for

$$L_{w,cg} = 3.75 + 5n \text{ cm}, n = 0 \dots 6.$$

Plot all seven cases on a common axis.

Modeling the pendulum

A schematic of the pendulum apparatus you'll be creating a numerical simulation for in Simulink and MATLAB is illustrated below. The system consists of three components: a rod with mass m_p and length L_p that is pivoted about a point near its top end; an attached circular weight of mass m_w and diameter d_w whose position along the rod is adjustable; and an attached sensor with diameter d_s. The fixed pivot axis passes though the center of the sensor, whose mass is neglected. The mass center locations for the rod and circular weight relative to the pivot point are denoted by $L_{p,cg}$ and $L_{w,cg}$, respectively.
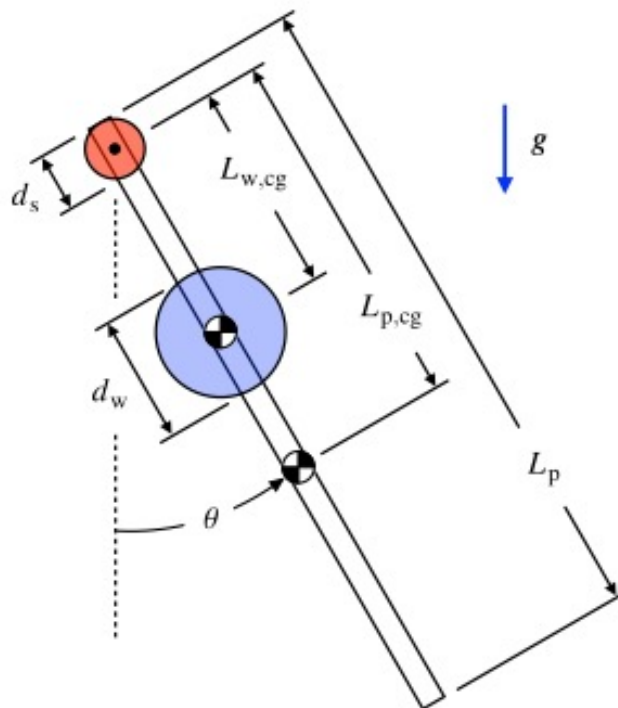
Applying the rate form of angular momentum conservation and kinematics, the equation of motion describing the pendulum's angular displacement θ measured counterclockwise from the downward position is

$$J\ddot{\theta} + k\sin\theta = 0 \ , \tag{1}$$

where the constants J and k are given by, respectively,

$$J = \frac{m_\mathrm{p}L_\mathrm{p}^2}{12} + m_\mathrm{p}L_\mathrm{p,cg}^2 + \frac{1}{2}m_\mathrm{w}\left(\frac{d_\mathrm{w}}{2}\right)^2 + m_\mathrm{w}L_\mathrm{w,cg}^2 \ , \tag{2}$$

$$k = g\left(m_\mathrm{p}L_\mathrm{p,cg} + m_\mathrm{w}L_\mathrm{w,cg}\right) \ . \tag{3}$$

The distance to the rod's mass center from the pivot point is calculated as

$$L_\mathrm{p,cg} = \frac{L_\mathrm{p} - d_\mathrm{s}}{2} \ . \tag{4}$$

Values for the various system parameters appearing in these equations are provided in the table below.

Parameter description	Symbol	Value with units
pendulum's mass	m_p	68.5 g
circular weight's mass	m_w	88 g
pendulum's length	L_p	43.2 cm
circular weight's diameter	d_w	5 cm
sensor's diameter	d_s	2.5 cm
gravitation acceleration	g	9.81 m/s^2

b) Numerical methods Review II: Modify your script from part a) to integrate Eq. (1) *backward* in time. Use a time step of 4 ms. Integrate backward in time from $\theta_0 = -\pi/2$ rad and $\dot{\theta}_0 = 1.5$ rad/s to find the time and position where $\dot{\theta} = 0$ rad/s. Repeat for the seven cases.
$$L_\mathrm{w,cg} = 3.75 + 5n \text{ cm}, n = 0 \dots 6.$$
Plot angular velocity for all seven cases on a common axis.

c) Numerical methods review III: One-dimensional Newton method for root finding.
The surface area of a right frustrum is given as:
$$A = \pi\left[\left(R_1^2 + R_2^2\right) + \sqrt{\left(R_1^2 - R_2^2\right)^2 + \left(h\left(R_1 + R_2\right)\right)^2}\right] \tag{5}$$

Write a well-documented **function** that, given area A, and radii of the base and apex (R_1 and R_2 respectively), uses the one-dimensional Newton method to find height h. Your **function** must accept the

dimensions R_1 and R_2, and area A as inputs and provide the height h, as the output. Your function must also print out the iteration information to the screen in the format shown below.

```
Iteration          height              function        derivative
-------------------------------------------------------------------
    1         x.xxxxxxxe-xxx         x.xxe+xxx        x.xxe-xxx
    2         x.xxxxxxxe-xxx         x.xxe+xxx        x.xxe-xxx
    .          . . . . . .            . . . .          . . . .
    .          . . . . . .            . . . .          . . . .
```

Review your notes from ME323/7 for the Newton root-finding algorithm.