

Build simulations of Extended and Unscented Kalman filters to estimate the time response of water height in a set of draining tanks, and the discharge coefficients.

A single water tank draining and open to the atmosphere at top and bottom can be modeled by the following first order ODE

$$\dot{h} = -C_D A \sqrt{(2g)} \sqrt{h + a} / A_0 \quad (1)$$

Where C_D is the discharge coefficient, A_0 is the tank cross-sectional area, A is the outlet cross-sectional area g is the gravity constant, h is the water height measured from the pressure transducer, and a is the nozzle length below the pressure transducer.

Many of the physical dimensions are illustrated in the Figure to the right. Pressure transducers measure the height of water in each tank with a linear mapping from water height to output voltage given by

$$v = mh + b$$

Where m is the transducer sensitivity in volts/inch, and b is the sensor bias in volts.

When the tanks drain simultaneously, the volumetric flow rates can be written as:

$$Q_1 = \begin{cases} C_{D1} A_1 \sqrt{(2g)} \sqrt{h_1 + a_1}, & h_1 > b_1 \\ 0, & \text{otherwise} \end{cases}$$

$$Q_2 = \begin{cases} C_{D2} A_2 \sqrt{(2g)} \sqrt{h_2 + a_2}, & h_2 > b_2 \\ 0, & \text{otherwise} \end{cases}$$

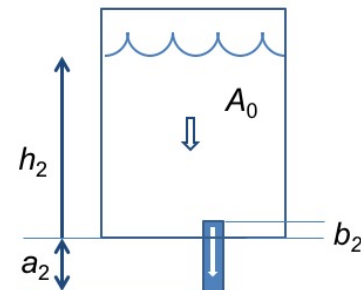
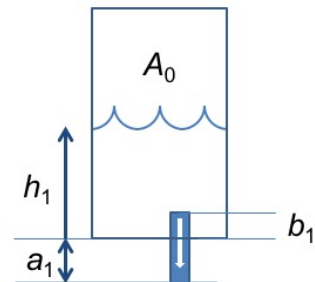


Figure 1: Cascaded draining tanks

the coupled equations can be written as

$$\dot{h}_1 = -\frac{Q_1}{A_0} \quad (2a)$$

$$\dot{h}_2 = \frac{Q_1 - Q_2}{A_0} \quad (2b)$$

For this project, you will build four simulations to estimate the water height(s) and discharge coefficients from a noisy experimental data set. Known parameters of the tanks and sensors are tabulated at the end of this handout. See the project 2 directory on Moodle for the data set. Parts a-d use the single tank data (`sys9_lower_ic.csv`). Part e uses the double tank data (`sys9_two_tank_ic.csv`).

- Write a Matlab code that employs the Extended Kalman Filter using the model in Eq. (1) to estimate the time history of the water height, and the discharge coefficient. Plot the time history of the filter estimate of discharge coefficient. Compare the filter estimate of height with the time history generated from the measurements using the known sensor parameters. Compare the filter measurement prediction history with the actual measurements.
- Modify your Extended Kalman Filter to also estimate the nozzle length (a). Use the dynamics Jacobian (\mathbf{A}) and measurement Jacobian (\mathbf{H}) to show that this filter will be unobservable.
- Write a Matlab code that uses the Unscented Kalman Filter with the model in Eq. (1) to estimate the time history of the water height, and the discharge coefficient. Plot the time history of the filter estimate of discharge coefficient. Compare the filter estimate of height with the time history generated from the measurements using the known sensor parameters. Plot the time history predicted measurement sigma points (\mathbf{Y}_k) and overlay the actual measurements.
- Write a Matlab code that uses the maximum likelihood method and batch updates to the initial height, discharge coefficient, and nozzle length, after predicting the full time response of Eq. (1), appending the state with sensitivities described by the following ODEs:

$$\frac{d}{dt} \left(\frac{\partial h}{\partial h_0} \right) = \frac{\partial \dot{h}}{\partial h} \frac{\partial h}{\partial h_0} = \frac{-C_D A \sqrt{(2g)}}{2A_0 \sqrt{h+a}} \cdot \frac{\partial h}{\partial h_0} \quad (3)$$

$$\frac{d}{dt} \left(\frac{\partial h}{\partial C_D} \right) = \frac{-A \sqrt{(2g)}}{A_0} \cdot \sqrt{h+a} \quad (4)$$

$$\frac{d}{dt} \left(\frac{\partial h}{\partial a} \right) = \frac{-C_D A \sqrt{(2g)}}{2A_0 \sqrt{h+a}} \quad (5)$$

Each iteration, use the costates, and predicted trajectory to form the Jacobian according to the normal equation: $\mathbf{J} \Delta \mathbf{w} = \mathbf{R}$ as shown in Eq. (6)

$$\begin{bmatrix} \left. \frac{\partial h}{\partial h_0} \right|_{t_1} & \left. \frac{\partial h}{\partial C_D} \right|_{t_1} & \left. \frac{\partial h}{\partial a} \right|_{t_1} \\ \left. \frac{\partial h}{\partial h_0} \right|_{t_2} & \left. \frac{\partial h}{\partial C_D} \right|_{t_2} & \left. \frac{\partial h}{\partial a} \right|_{t_2} \\ \vdots & \vdots & \vdots \end{bmatrix} \begin{Bmatrix} \Delta h_0 \\ \Delta C_D \\ \Delta a \end{Bmatrix} = \begin{Bmatrix} h(t_1) - h_{dat}(t_1) \\ h(t_2) - h_{dat}(t_2) \\ \vdots \end{Bmatrix} \quad (6)$$

Then the parameters changes are determined using the maximum likelihood update:

$$\Delta \mathbf{w} = (\mu \mathbf{I} + \mathbf{J}^T \mathbf{J})^{-1} \mathbf{J}^T \mathbf{R} \quad (7)$$

And the parameter vector is updated by $\mathbf{w}^{\text{new}} = \mathbf{w}^{\text{old}} - \Delta \mathbf{w}$. Continue the iteration until $\|\Delta \mathbf{w}\|_2$ is less than 10^{-6} .

- e) Write a Matlab code that uses the Unscented Kalman Filter to estimate the time history of water heights and discharge coefficients for the two-tank system, using the model in Eqs. (2a-b). Plot the time history of the filter estimates of discharge coefficients. Compare the filter estimates of height with the time history generated from the measurements using the known sensor parameters. Plot the time history predicted measurement sigma points (\mathbf{Y}_k) and overlay the actual measurements.

Data files: Parts a) through d) -> use '[sys9_lower_ic.csv](#)' and the tank 2 parameters below
Part e) -> use '[sys9_two_tank_ic.csv](#)' and the all tank parameters below

Table 1: Known Tank Parameters

Tank	a_i (in)	d_i (in)	A_0 (in ²)	m_i (V/in)	b_i (V)
1 (upper)	3.375	0.359	24.016	0.11927	0.978
2 (lower)	3.375	0.237	24.016	0.12279	1.2467